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Consistent estimation of rational expectation models

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CONSISTENT ESTIMATION OF RATIONAL
EXPECTATIONS MODELS

T.E. Nijman and F.C. Palm

CONSISTENT ESTIMATION OF RATIONAL

===== EXPECTATIONS MODELS =====

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T.E. Nijman^{*} and F.C. Palm^{**}

February 1986

A B S T R A C T

We discuss the use of proxy variables in consistent estimation of the parameters of rational expectations models. The estimators considered are more robust and computationally less demanding than the maximum likelihood estimator.

To provide some guidance for choosing the proxy variables and the estimator, we propose a consistent generalized least squares estimator and show that it is asymptotically more efficient than alternative estimators based on approximations for the unobserved expectations. Numerical results for several simple rational expectations models illustrate the relative efficiency of various proxy variables estimators.

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1. Introduction

Models with expectational variables are widely applied in empirical econometric research. Various estimation methods have been put forward for these models. Some methods are based on proxy variables which are substituted for the unobserved expectations. The resulting model is subsequently estimated by e.g. an instrumental variables (IV) method. References to proxy variables estimators in the rational expectations literature are McCallum (1976), Sargent (1976), Barro (1977) and Pagan (1984) among others.

In this paper, we are concerned with the efficiency of proxy variables estimators for rational expectations models. Although proxy variables estimators are usually not fully efficient, they have several advantages compared with efficient estimation methods such as e.g. Maximum Likelihood (ML) estimators. For instance, they often do not require a fully specified model. Therefore, they are expected to be more robust with respect to specification uncertainty than full information methods. Moreover, they are often computationally more attractive than ML estimators.

The paper is organized as follows. In section 2, we state the main result and show that it can be used as a guidance to increase the efficiency of a proxy variables estimator (PVE). In section 3, numerical results on the relative efficiency of PVE's illustrate the result of section 2 for a model with rational expectations. Finally, section 4 contains some concluding remarks.

2. Efficiency of proxy variables estimators

Consider the following linear model

$$\begin{matrix} y \\ T \times 1 \end{matrix} = \begin{matrix} X\beta \\ T \times k \quad k \times 1 \end{matrix} + \begin{matrix} \epsilon, \\ T \times 1 \end{matrix} \quad (2.1)$$

where y and X are the endogenous and the explanatory variables respectively, β is the vector of coefficients, ϵ is assumed to be normally distributed with mean zero and covariance matrix $\sigma^2 I$, and T is the sample size.

Suppose that $X = [x_1^e \ x_2]$ where x_1^e is a column vector of unobserved expectations. Moreover assume that the typical element x_{1t}^e of x_1^e linearly depends on a finite number of observed variables

$$\begin{aligned} x_1^e &= Z\alpha. \\ Tx1 & \quad Tx2 \quad \{x1 \end{aligned} \tag{2.2}$$

Frequently, in particular when (2.1) is part of a larger model, restrictions on (α, β) will be available. However, for the reasons outlined in the introduction, we shall disregard these restrictions here. Finally, we assume that $\text{plim } T^{-1}x_2'\epsilon = 0$ and $\text{plim } T^{-1}Z'\epsilon = 0$. If we substitute the vector of realized values x_1 as a proxy variable for x_1^e in (2.1), as suggested by McCallum (1976), we obtain

$$\begin{aligned} y &= \tilde{X}\beta + \tilde{w}, \quad \tilde{X} = [x_1 \ x_2] \\ \tilde{w} &= \epsilon + (x_1^e - x_1)\beta_1, \end{aligned} \tag{2.3}$$

with β_1 being the first element of β . Of course, the use of x_1 as a proxy for x_1^e in (2.1) requires that $y \neq x_1$ and that x_1 is not a column of x_2 . The vector β in (2.1) can be consistently estimated from (2.3) by instrumental variables

$$\hat{\beta}_{IV} = (Z^*'\tilde{X})^{-1} Z^*y, \tag{2.4}$$

with $Z^* = Z(Z'Z)^{-1} Z'\tilde{X}$, provided $\text{plim } T^{-1}Z^*\tilde{X}$ is a finite nonsingular matrix.

A second proxy for x_1^e in (2.1) that has been proposed in the literature can be obtained by estimating α in

$$x_1 = Z\alpha + u \tag{2.5}$$

by OLS yielding $\hat{\alpha}$ and using $\hat{x}_1^e = Z\hat{\alpha}$ as an approximation for x_1^e in (2.1). If we define $\hat{X} = [Z\hat{\alpha} \ x_2]$, we get

$$y = \hat{X}\beta + \hat{w} \tag{2.6}$$

with $\hat{w} = \epsilon + \beta_1 Z(\alpha - \hat{\alpha})$. The vector β in (2.6) can be consistently estimated by OLS

$$\hat{\beta}_{OLS} = (\hat{X}'\hat{X})^{-1} \hat{X}'y. \quad (2.7)$$

This estimator coincides with $\hat{\beta}_{IV}^{\gamma}$ if X_2 is included in Z . From (2.6), it is apparent that there is no need to instrument X_2 in (2.3). Therefore, an efficiency gain might be expected from the use of $\hat{\beta}_{OLS}$ if X_2 is not in Z as might for instance be the case in modeling futures markets or long term contracts. However $\hat{\beta}_{OLS}$ is not necessarily more efficient than $\hat{\beta}_{IV}^{\gamma}$ as the elements of \hat{w} are autocorrelated. This leaves us with the choice among proxy variables and instruments. The next theorem provides some guidance on how to select the proxies and the instruments to increase the efficiency of the estimator of the parameters β .

Theorem

Assume that (2.1) holds with $\text{plim } T^{-1}Z'\epsilon = 0$ and \hat{X} and \tilde{X} are two proxies for X . Consider the estimators

$$\hat{\beta}_{GLS} = (\hat{X}'\hat{\Sigma}^{-1}\hat{X})^{-1} \hat{X}'\hat{\Sigma}^{-1}y \text{ and } \hat{\beta}_{IV}^{\gamma} = (Z'\tilde{X})^{-1} Z'y. \quad (2.8)$$

Assume that

$$(i) \quad \sqrt{T} (\hat{\beta}_{GLS} - \beta) \underset{a}{\sim} N(0, V^{-1}), \text{ where } V^{-1} = \text{plim } T(\hat{X}'\hat{\Sigma}^{-1}\hat{X})^{-1}$$

is finite and positive definite;

$$(ii) \quad \text{plim } T^{-1}Z'\tilde{X} = Q \text{ is finite and positive definite;}$$

$$(iii) \quad \sqrt{T} \begin{bmatrix} Z'\hat{w} \\ Z'(\tilde{w} - \hat{w}) \end{bmatrix} \underset{a}{\sim} N(0, D), \text{ where } D = \text{plim } T^{-1} \begin{bmatrix} Z'\hat{\Sigma}^{-1}Z & 0 \\ 0 & Z'SZ \end{bmatrix}$$

for some S , and \hat{w} and \tilde{w} are the disturbances associated with \hat{X} and \tilde{X} respectively.

Then $\hat{\beta}_{GLS}$ is asymptotically at least as efficient as $\hat{\beta}_{IV}^{\gamma}$.

Proof : see appendix A.

The third requirement is most crucial. If two proxies \hat{X} and \tilde{X} are available, an IV estimator based on \tilde{X} cannot be more efficient than a GLS estimator

based on \hat{X} if $Z'\hat{w}$ and $Z'(\hat{w} - \tilde{w})$ are asymptotically orthogonal, provided the regularity conditions of the theorem are met. The theorem can be used to demonstrate the relative efficiency of GLS estimators for various types of models. For models with imputed data sets in the case of missing observations, we refer to Nijman and Palm (1984) and Nijman (1985).

Now we discuss the relative asymptotic efficiency of some consistent estimators of β in (2.1). Equation (2.6) can be consistently estimated by the feasible GLS estimator $\hat{\beta}_{GLS}$ in (2.8), where $\hat{\Sigma}^{-1}$ is chosen such that assumption (i) of the theorem is satisfied. For instance, if ϵ and u are independent, a matrix $\hat{\Sigma}^{-1}$ which satisfies this assumption is

$$\hat{\Sigma}^{-1} = \hat{\sigma}^{-2} I - \hat{\beta}_1^2 \hat{\sigma}^{-2} Z \{ \hat{\sigma}^2 R + \hat{\beta}_1^2 (Z'Z) \}^{-1} Z', \quad (2.9)$$

with R , $\hat{\sigma}^2$ and $\hat{\beta}_1$ being consistent estimates of the asymptotic covariance matrix of $\hat{\alpha}$, and of the parameters σ^2 and β_1 respectively.

The well known result that the feasible GLS estimator is asymptotically at least as efficient as OLS, which we denote as $\hat{\beta}_{GLS}(Z) \geq \hat{\beta}_{OLS}(Z)$, also follows from the theorem. The argument between parentheses indicates the variables on which the proxy for x_1^e is based.

The theorem implies that GLS is at least as efficient as the IV estimator. Moreover it can be used to show that GLS is at least as efficient as the estimator put forward by Cumby, Huizinga and Obstfeld (1983), hereafter denoted as CHO, which reads as follows

$$\hat{\beta}_{CHO} = [\hat{X}'Z \hat{\Omega}_Z^{-1} Z'\hat{X}]^{-1} \hat{X}'Z \hat{\Omega}_Z^{-1} Z'y, \quad (2.10)$$

with $T^{-1} \hat{\Omega}_Z$ being a consistent estimate of $\text{plim } T^{-1} Z'\Omega Z$ and $\Omega = E \tilde{w} \tilde{w}'$ in (2.3). The proof is based on the fact that

$$Z'(\tilde{w} - \hat{w}) = Z'(\epsilon - \beta_1 u) - Z'(\epsilon - \beta_1 Z(Z'Z)^{-1} Z'u) = 0.$$

The theorem can also be used to assess the effect of the inclusion of additional regressors in (2.5) on the asymptotic variance of the GLS estimator. If (2.9) applies, the GLS estimator uses a linear combination of X_2 and Z as instruments. Hence the theorem implies that

$$\hat{\beta}_{GLS}(Z^{(1)}) \geq \hat{\beta}_{GLS}(Z^{(2)}) \text{ where } Z^{(2)} = (Z^{(1)}, Z^{(12)}) \text{ if } H'(\tilde{w}^{(1)} - \tilde{w}^{(2)})$$

is asymptotically orthogonal to $H'w^{(1)}$ with $w^{(i)} = \epsilon - \beta_1 z^{(i)} [Z^{(i)'} Z^{(i)}]^{-1} Z^{(i)'} u$ and $H = [Z^{(2)} X_2]$. The inclusion of regressors in (2.5) which have zero coefficients cannot improve the efficiency of the GLS estimator as $H'(w^{(1)} - w^{(2)})$ is asymptotically orthogonal to $H'w^{(1)}$ as a direct consequence of the assumptions about ϵ and u and the properties of iterated expectations.

The i -th elements of $T^{-\frac{1}{2}} H'(w^{(1)} - w^{(2)})$ and $T^{-\frac{1}{2}} H'w^{(1)}$ converge to $T^{-\frac{1}{2}} \beta_1 \sum_t (h_{it}^{(2)} - h_{it}^{(1)}) u_t$ and $T^{-\frac{1}{2}} \sum_t h_{it}^{(1)} (\beta_1 u_t + \epsilon_t)$ respectively with $h_{it}^{(k)} = E(h_{it} | z_t^{(k)})$ being the projection of h_{it} on $z_t^{(k)}$. For the asymptotic covariances we have $E\{T^{-1} \beta_1^2 \sum_{t,s} (h_{it}^{(2)} - h_{it}^{(1)}) u_t u_s h_{js}^{(1)}\} = E\{T^{-1} \beta_1^2 \sum_{t,s} E[(h_{it}^{(2)} - h_{it}^{(1)}) h_{js}^{(1)} | z_t^{(1)}] E(u_t u_s | z_t^{(1)})\}$, where the first factor of the r.h.s. terms equals zero. An analogous result applies to cross moments between u_t and ϵ_s . Similar results can be obtained for more general models ¹⁾.

Notice that on the contrary, the efficiency of $\hat{\beta}_{IV}$ and $\hat{\beta}_{CHO}$ usually increases when Z in (2.5) is expanded. This also holds true for the estimator proposed by Hayashi and Sims (1983), hereafter denoted as HS, which reads as

$$\hat{\beta}_{HS} = [\hat{X}' P^{-1} Z(Z'Z)^{-1} Z' P^{-1} \hat{X}]^{-1} \hat{X}' P^{-1} Z(Z'Z)^{-1} Z' P^{-1} y, \quad (2.11)$$

where P is a consistent estimate of an upper-triangular matrix Π satisfying $\Omega = \Pi \Pi'$. Notice also that CHO (1983) and HS (1983) show respectively that $\hat{\beta}_{CHO}(Z) > \hat{\beta}_{IV}(Z)$ and $\hat{\beta}_{HS}(Z) > \hat{\beta}_{IV}(Z)$.

If $H'(w^{(2)} - w^{(1)})$ is not only asymptotically orthogonal to $H'w^{(1)}$ but to $H'w^{(2)}$ as well, $\hat{\beta}_{GLS}(Z^{(1)})$ and $\hat{\beta}_{GLS}(Z^{(2)})$ are equally efficient. If $E(h_t | z_t^{(1)}) = E(h_t | z_t^{(2)})$ or in short $h_t^{(1)} = h_t^{(2)}$, $T^{-\frac{1}{2}} H'(w^{(2)} - w^{(1)})$ converges to zero in probability so that it is asymptotically orthogonal to every other variable.

To illustrate this result, define \bar{Z} as the matrix of variables with nonzero coefficients in (2.5). This matrix is the minimal regressor matrix which assures consistency of $\hat{\beta}_{OLS}$ and $\hat{\beta}_{GLS}$. If $\hat{\Sigma}^{-1}$ has the structure as outlined in footnote (1), the inclusion of all variables in \bar{Z} lagged one period will in general lead to an efficiency loss as $h_t^{(1)} \neq h_t^{(2)}$, because of the presence of \bar{z}_{t-1} in h_t , $E(\bar{z}_{t-1} | \bar{z}_t) \neq E(\bar{z}_{t-1} | \bar{z}_t, \bar{z}_{t-1})$. If on the other hand (2.9) applies, the condition $E(x_{2t} | \bar{z}_t) = E(x_{2t} | \bar{z}_{t-1}, i=0,1,\dots)$ implies that subsequent addition of \bar{z}_{t-i} , $i=1,2,\dots$ to the regressors in

(2.5) does not affect the asymptotic efficiency of $\hat{\beta}_{GLS}$.

To summarize, we conclude that the efficiency of $\hat{\beta}_{GLS}$ can sometimes decrease if the dimension of Z in (2.5) is increased. This does not occur for $\hat{\beta}_{IV}$, $\hat{\beta}_{CHO}$ and $\hat{\beta}_{HS}$.

HS (1983) have shown that the difference between the asymptotic variances of $\hat{\beta}_{CHO}$ and $\hat{\beta}_{HS}$ converges to zero if the \bar{z}_{t-i} , $i=1,2,\dots$, are subsequently added to the regressors in (2.5). This result can be combined with those given above to yield $\hat{\beta}_{GLS}(Z) > \hat{\beta}_{HS}(Z)$.

Finally, we consider the use of alternative estimates of α in (2.2).

Until now, we restricted ourselves to the use of the OLS estimator of α in (2.7). An example where two consistent estimators of α , $\hat{\alpha}$ and $\tilde{\alpha}$, are available is given in the next section. If $\hat{\alpha}$ is efficient within the class of estimators containing all linear combinations of $\hat{\alpha}$ and $\tilde{\alpha}$, and ε is independent of $\tilde{\alpha}$ and $\hat{\alpha}$, the theorem implies that the GLS estimator based on $\hat{\alpha}$ will be more efficient than the one based on $\tilde{\alpha}$.

The orthogonality condition is then satisfied as $H'Z(\hat{\alpha} - \tilde{\alpha})$ is orthogonal to $H'\varepsilon + H'Z(\alpha - \hat{\alpha})$ as a result of the orthogonality in large samples between $\sqrt{T}(\hat{\alpha} - \tilde{\alpha})$ and $\sqrt{T}(\alpha - \tilde{\alpha})$ (for this orthogonality condition see e.g. Hausman (1978)). However if ε is not independent of $\tilde{\alpha}$ and $\hat{\alpha}$, more efficient estimation of α does not necessarily lead to more efficient GLS estimation of β .

3. Numerical results on the relative efficiency

In this section, numerical results on the relative asymptotic efficiency of the estimators presented in the previous section will be given. We consider the following models

$$y_t = \beta_1 \eta_t^e + \beta_2 x_t + \varepsilon_t; \varepsilon_t \sim IN(0, \sigma_\varepsilon^2), \quad (3.1)$$

$$x_t = \gamma_1 x_{t-1} + \gamma_2 x_{t-2} + v_t; v_t \sim IN(0, \sigma_v^2), \quad (3.2)$$

where ε_t and v_s are independent for all t and s and where for the expectation η_t^e is defined as

$$\eta_t^e = E(y_{t+1} | I_t) \quad \text{for model I,}$$

$$\eta_t^e = E(y_{t+1} | I_{t-1}) \quad \text{for model II,}$$

$$\eta_t^e = E(x_t | I_t) \quad \text{for model III, and}$$

$$\eta_t^e = E(x_{t+1} | I_{t-1}) \quad \text{for model IV,}$$

with $I_t = \{y_t, x_t, y_{t-1}, x_{t-1}, \dots\}$. Although these models are simple, they illustrate important features of more realistic models quite well.

Model I has been analyzed by Gouriéroux et al. (1982), to whom we refer for a justification of its use. Model II arises in the analysis of futures markets and long term contracts.

For the models I and III, $\eta_t^e = \alpha_{i1}x_t + \alpha_{i2}x_{t-1}$ and for the models II and IV, $\eta_t^e = \alpha_{i1}x_{t-1} + \alpha_{i2}x_{t-2}$, where the index i refers to model i . If we define $x_{1t} = y_{t+1}$ in models I and II and $x_{1t} = x_{t+1}$ in models III and IV, u_t in (2.5) is given by

$$u_t = \varepsilon_{t+1} + (\beta_2 + \beta_1\alpha_{11})v_{t+1} \quad \text{in model I,}$$

$$u_t = \varepsilon_{t+1} + \beta_2v_{t+1} + (\beta_2\gamma_1 + \beta_1\alpha_{21})v_t \quad \text{in model II,} \quad (3.3)$$

$$u_t = v_{t+1} \quad \text{in model III,}$$

$$u_t = v_{t+1} + \gamma_1v_t \quad \text{in model IV.}$$

Note that in order to compute the proxy variables estimators, there is no need to derive the α 's explicitly as functions of the parameters in (3.1) and (3.2). For model IV, we could estimate γ_1 and γ_2 from (3.2) by OLS, then construct an estimate of the two-step ahead predictor of x_{t+1} given I_{t-1} , substitute it for η_t^e in (3.1) and estimate (3.1) by GLS. This estimator is fully efficient in the present case. This example illustrates the gain of efficiency due to the use of a more efficient estimate of α in (2.2).

The expressions for u_t in (3.3) imply that in models III and IV the feasible GLS estimator can be obtained using (2.9) while for models I and II, the expression for $\hat{\Sigma}^{-1}$ given in footnote (1) applies. For the models I and III,

$\hat{\beta}_{OLS}$ coincides with $\hat{\beta}_{IV}^{\gamma}$. For models II and IV however, x_t is projected on x_{t-1} and x_{t-2} in the computation of $\hat{\beta}_{IV}^{\gamma}$, $\hat{\beta}_{CHO}^{\gamma}$ and $\hat{\beta}_{HS}^{\gamma}$ but not in that of $\hat{\beta}_{OLS}$ and $\hat{\beta}_{GLS}$. One might expect that for these models, $\hat{\beta}_{OLS}$ is more efficient than the estimators based on (2.3) and that the efficiency gain of $\hat{\beta}_{GLS}$ over these estimators is more important for the models II and IV than for the models I and III.

The efficiency of the various estimators of β relative to that of the maximum likelihood estimator is measured by the ratio of the asymptotic variances. The results for β_1 are presented in table 1 for the models I, II and IV. Results for β_2 are given in table 2 in appendix B. The values of the parameters are also given in the tables.

The derivation of the results is given in appendix C. For model III, all estimators are fully efficient, a property which is also proved in appendix C.

Six proxy variables estimators have been considered. The estimators $\hat{\beta}_{OLS}^{\gamma}$, $\hat{\beta}_{GLS}^{\gamma}$ and $\hat{\beta}_{IV}^{\gamma}$ are based on the minimal set of instruments, that is the typical row of Z is (x_t, x_{t-1}) for model I and (x_{t-1}, x_{t-2}) for models II and IV. For the estimators $\hat{\beta}_{GLS2}^{\gamma}$, $\hat{\beta}_{CHO}^{\gamma}$ and $\hat{\beta}_{HS}^{\gamma}$, Z contains the observations on (x_t, x_{t-1}, x_{t-2}) for model I and $(x_{t-1}, x_{t-2}, x_{t-3})$ for models II and IV. It can be easily verified that the relative efficiency of the estimators depends on β_1 , γ_1 , γ_2 and R^2 in the models I and II, with

$$R^2 = E(y_t - \varepsilon_t)^2 E^{-1} y_t^2. \quad \text{In model IV, the relative efficiency depends on } \beta_1 \beta_2^{-1}, R^2, \gamma_1 \text{ and } \gamma_2.$$

From table 1 it is apparent that for model I all estimators are almost as efficient as ML if $|\beta_1|$ is small. For larger values of β_1 , the differences between the various estimators compared to ML are more pronounced, except for $\hat{\beta}_{GLS}$ which is fully efficient here. For models II and IV, the efficiency loss of the estimators based on the substitution of the realization for the unobserved expectations (as suggested e.g. by McCallum (1976)) can be very large when $|\gamma_2|$ is small. This is not surprising as in models II and IV only these estimators lose their consistency if γ_2 is zero as then a multicollinearity problem arises. The estimator $\hat{\beta}_{OLS}^{\gamma}$ is sometimes less efficient than $\hat{\beta}_{CHO}^{\gamma}$ or $\hat{\beta}_{HS}^{\gamma}$. The relative efficiency of $\hat{\beta}_{GLS2}^{\gamma}$ is smaller than that of $\hat{\beta}_{GLS}^{\gamma}$ for model I and II. This illustrates the loss of efficiency due to the use of extra instruments, in addition to those in the information sets of the rational expectations. The results for β_2 reported in table 2

are similar and lead to the same conclusions as for β_1 . Notice finally that if the lag structure of the auxiliary equation (3.2) is misspecified, $\hat{\beta}_{ML}$ will no longer be consistent. The PVEs however are robust to such misspecifications. Nevertheless, for $\hat{\beta}_{OLS}$ and $\hat{\beta}_{GLS}$, a misspecified equation (3.2) can easily lead to inconsistent estimates of asymptotic standard errors.

4. Conclusions

In this paper we derived a theorem which can be used to compare the efficiency of consistent PVEs. We discussed the implications of the theorem for the estimation of models with unobserved expectations. Numerical results illustrated these implications for several simple models with rational expectations. It appeared that some PVEs are not only more robust and computationally more attractive than the ML estimator but almost asymptotically efficient as well in the models considered. Some instrumental variables estimators using the realizations as proxies for the unobserved expectations appeared to be very inefficient in several of the cases that we have analyzed. Therefore, their use in applied work cannot be recommended.

Table 1 : Relative efficiency of the maximum likelihood estimator compared with alternative estimators for β_1 , measured by the ratio of large sample variances.

Model I									
β_1	R^2	Y_1	Y_2	$\hat{\beta}_{OLS}$	$\hat{\beta}_{GLS}$	$\hat{\beta}_{GLS2}$	$\hat{\beta}_{IV}$	$\hat{\beta}_{CHO}$	$\hat{\beta}_{HS}$
1.20	0.50	0.80	-0.15	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.20	-0.27	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.20	-0.35	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.60	-0.63	1.03	1.00	1.00	1.03	1.00	1.00
0.20	0.90	0.80	-0.15	1.03	1.00	1.01	1.03	1.01	1.01
0.20	0.90	1.20	-0.27	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.90	1.20	-0.35	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.90	1.60	-0.63	1.03	1.00	1.00	1.03	1.00	1.00
0.80	0.50	0.80	-0.15	1.51	1.00	1.14	1.51	1.26	1.20
0.80	0.50	1.20	-0.27	1.80	1.00	1.07	1.80	1.34	1.14
0.80	0.50	1.20	-0.35	1.72	1.00	1.12	1.72	1.33	1.20
0.80	0.50	1.60	-0.63	2.21	1.00	1.06	2.21	1.50	1.13
0.80	0.90	0.80	-0.15	1.15	1.00	1.11	1.15	1.12	1.12
0.80	0.90	1.20	-0.27	1.39	1.00	1.18	1.39	1.23	1.21
0.80	0.90	1.20	-0.35	1.23	1.00	1.15	1.23	1.17	1.16
0.80	0.90	1.60	-0.63	1.52	1.00	1.17	1.52	1.27	1.22
Model II									
0.20	0.50	0.80	-0.15	1.01	1.00	1.00	157.23	150.58	150.27
0.20	0.50	1.20	-0.27	1.00	1.00	1.00	102.04	98.07	97.88
0.20	0.50	1.20	-0.35	1.00	1.00	1.00	21.27	20.45	20.42
0.20	0.50	1.60	-0.63	1.02	1.00	1.00	3.07	2.99	2.98
0.20	0.90	0.80	-0.15	1.01	1.00	1.00	156.32	145.07	144.19
0.20	0.90	1.20	-0.27	1.00	1.00	1.00	106.34	101.37	101.14
0.20	0.90	1.20	-0.35	1.01	1.00	1.00	22.39	21.20	21.13
0.20	0.90	1.60	-0.63	1.02	1.00	1.00	3.08	2.98	2.98
0.80	0.50	0.80	-0.15	1.06	1.00	1.00	146.89	106.01	82.26
0.80	0.50	1.20	-0.27	1.02	1.00	1.00	96.35	69.89	57.00
0.80	0.50	1.20	-0.35	1.08	1.00	1.01	15.47	11.25	9.47
0.80	0.50	1.60	-0.63	1.80	1.00	1.08	2.61	1.75	1.32
0.80	0.90	0.80	-0.15	1.06	1.04	1.04	129.53	95.56	78.98
0.80	0.90	1.20	-0.27	1.02	1.00	1.00	139.57	116.39	111.57
0.80	0.90	1.20	-0.35	1.10	1.00	1.02	23.02	19.61	19.02
0.80	0.90	1.60	-0.63	1.22	1.00	1.11	2.12	1.76	1.68
Model IV									
$\beta_1 \beta_2^{-1}$	R^2	Y_1	Y_2	$\hat{\beta}_{OLS}$	$\hat{\beta}_{GLS}$	$\hat{\beta}_{GLS2}$	$\hat{\beta}_{IV}$	$\hat{\beta}_{CHO}$	$\hat{\beta}_{HS}$
0.50	0.50	0.80	-0.15	1.01	1.00	1.00	167.46	166.75	166.74
0.50	0.50	1.20	-0.27	1.00	1.00	1.00	121.84	121.86	121.85
0.50	0.50	1.20	-0.35	1.00	1.00	1.00	27.61	27.59	27.59
0.50	0.50	1.60	-0.63	1.00	1.00	1.00	4.43	4.43	4.43
0.50	0.90	0.80	-0.15	1.07	1.05	1.05	182.66	165.15	165.11
0.50	0.90	1.20	-0.27	1.00	1.00	1.00	136.59	135.10	135.08
0.50	0.90	1.20	-0.35	1.01	1.01	1.01	31.19	30.20	30.20
0.50	0.90	1.60	-0.63	1.01	1.00	1.00	4.47	4.47	4.47
2.00	0.50	0.80	-0.15	1.06	1.05	1.05	180.10	165.80	165.83
2.00	0.50	1.20	-0.27	1.00	1.00	1.00	128.26	127.85	127.89
2.00	0.50	1.20	-0.35	1.01	1.01	1.01	29.49	29.12	29.12
2.00	0.50	1.60	-0.63	1.00	1.00	1.00	4.45	4.45	4.45
2.00	0.90	0.80	-0.15	1.24	1.19	1.19	215.47	148.86	146.58
2.00	0.90	1.20	-0.27	1.01	1.00	1.00	183.13	167.74	167.78
2.00	0.90	1.20	-0.35	1.04	1.03	1.03	39.90	32.74	32.70
2.00	0.90	1.60	-0.63	1.04	1.01	1.01	4.66	4.62	4.62

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1) FOOTNOTE

For instance, in the case of future expectations in (2.1), it often holds true that $EEu' = \beta_1 \sigma^2 L$, where L is a matrix with zero elements, but for the $(i+1, i)$ -th positions ($i=1, \dots, T-1$), which are equal to one. Then a matrix $\hat{\Sigma}^{-1}$ which satisfies assumption (i) of the theorem is given by

$$\hat{\Sigma}^{-1} = S_1 - S_2 S_3^{-1} S_2' \quad \text{with } S_1 = \hat{\sigma}^{-2} I + \hat{\beta}_1^2 Z_{-1}' W^{-1} Z_{-1},$$

$$S_2 = \hat{\sigma}^{-2} I - \hat{\beta}_1 Z_{-1}' W^{-1} R, \quad S_3 = \hat{\sigma}^{-2} Z'Z + R'W^{-1}R, \quad R = Z'Z - \hat{\beta}_1 Z'Z_{-1},$$

where W is a consistent estimate of $M - \sigma^2 \text{plim } T^{-1} Z'Z$, where M is the asymptotic covariance matrix of $T^{-\frac{1}{2}} Z'u$ and Z_{-1} is a matrix which contains one period lagged values of Z . In this case, $\hat{\beta}_{\text{GLS}}$ uses a linear combination of X , Z and Z_{-1} as instruments and H above has to be redefined as $H = [X_2, Z, Z_{-1}]$. Again, the inclusion of regressors in (2.5) which have zero coefficients does not increase the efficiency of the GLS estimator.

APPENDIX A : Proof of the theorem

Using assumptions (ii) and (iii) one verifies that

$$\sqrt{T}(\hat{\beta}_{IV} - \beta) \underset{a}{\sim} N(0, Q^{-1}(D_{11} + D_{22})Q^{-1}), \text{ where } D_{11} \text{ and } D_{22}$$

are the upper-left and the lower-right blocks of D respectively. Furthermore, the asymptotic orthogonality of Z and ε and assumption (iii) imply that

$$\text{plim } T^{-1}Z'(\hat{w} - \hat{w}) = \text{plim } T^{-1}Z'(\hat{X} - \hat{X})\beta = 0,$$

for all β, so that $\text{plim } T^{-1}Z'\hat{X} = \text{plim } T^{-1}Z'\hat{X} = Q$. Using this result, one obtains that

$$\begin{aligned} Q^{-1}(D_{11} + D_{22})Q^{-1} &= \text{plim } T^{-1}(Q^{-1}Z' - V^{-1}\hat{X}'\hat{\Sigma}^{-1})\hat{\Sigma}^{-1}(ZQ^{-1} - \hat{\Sigma}^{-1}\hat{X}V^{-1}) \\ &\quad + \text{plim } T^{-1}Q^{-1}Z'\hat{X}V^{-1} + \text{plim } T^{-1}V^{-1}\hat{X}'ZQ^{-1} \\ &\quad - V^{-1} + Q^{-1}D_{22}Q^{-1} \geq V^{-1} \end{aligned}$$

which proves the result.

APPENDIX B : Table 2 : Relative efficiency of the maximum likelihood estimator compared with alternative estimators for β_2 , measured by the ratio of large sample variances.

Model I									
β_1	R^2	γ_1	γ_2	$\hat{\beta}_{OLS}$	$\hat{\beta}_{GLS}$	$\hat{\beta}_{GLS2}$	$\hat{\gamma}_{IV}$	$\hat{\gamma}_{CHO}$	$\hat{\gamma}_{HS}$
0.20	0.50	0.80	-0.15	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.20	-0.27	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.20	-0.35	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.50	1.60	-0.63	1.03	1.00	1.00	1.03	1.00	1.00
0.20	0.90	0.80	-0.15	1.03	1.00	1.01	1.03	1.01	1.01
0.20	0.90	1.20	-0.27	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.90	1.20	-0.35	1.04	1.00	1.00	1.04	1.00	1.00
0.20	0.90	1.60	-0.63	1.03	1.00	1.00	1.03	1.00	1.00
0.80	0.50	0.80	-0.15	1.58	1.00	1.16	1.58	1.29	1.22
0.80	0.50	1.20	-0.27	1.85	1.00	1.08	1.85	1.36	1.15
0.80	0.50	1.20	-0.35	1.82	1.00	1.14	1.82	1.37	1.22
0.80	0.50	1.60	-0.63	2.30	1.00	1.07	2.30	1.53	1.14
0.80	0.90	0.80	-0.15	1.15	1.00	1.11	1.15	1.12	1.12
0.80	0.90	1.20	-0.27	1.39	1.00	1.19	1.39	1.24	1.22
0.80	0.90	1.20	-0.35	1.23	1.00	1.15	1.23	1.16	1.16
0.80	0.90	1.60	-0.63	1.51	1.00	1.17	1.51	1.27	1.22
Model II									
0.20	0.50	0.80	-0.15	1.00	1.00	1.00	290.00	277.73	277.18
0.20	0.50	1.20	-0.27	1.00	1.00	1.00	109.71	105.45	105.24
0.20	0.50	1.20	-0.35	1.00	1.00	1.00	24.72	23.78	23.74
0.20	0.50	1.60	-0.63	1.02	1.00	1.00	3.12	3.03	3.03
0.20	0.90	0.80	-0.15	1.00	1.00	1.00	347.96	322.78	320.80
0.20	0.90	1.20	-0.27	1.00	1.00	1.00	115.68	110.28	110.02
0.20	0.90	1.20	-0.35	1.00	1.00	1.00	26.86	25.45	25.37
0.20	0.90	1.60	-0.63	1.02	1.00	1.00	3.12	3.03	3.02
0.80	0.50	0.80	-0.15	1.00	1.00	1.00	336.36	238.78	182.08
0.80	0.50	1.20	-0.27	1.01	1.00	1.00	99.70	72.04	58.56
0.80	0.50	1.20	-0.35	1.04	1.00	1.02	17.35	12.45	10.38
0.80	0.50	1.60	-0.63	1.85	1.00	1.08	2.64	1.76	1.31
0.80	0.90	0.80	-0.15	1.02	1.00	1.00	954.04	693.79	566.78
0.80	0.90	1.20	-0.27	1.01	1.00	1.00	163.40	136.11	130.43
0.80	0.90	1.20	-0.35	1.14	1.00	1.00	39.27	33.43	32.41
0.80	0.90	1.60	-0.63	1.20	1.00	1.10	2.10	1.75	1.68
Model IV β_1, β_2^{-1}	R^2	γ_1	γ_2	$\hat{\beta}_{OLS}$	$\hat{\beta}_{GLS}$	$\hat{\beta}_{GLS2}$	$\hat{\gamma}_{IV}$	$\hat{\gamma}_{CHO}$	$\hat{\gamma}_{HS}$
0.50	0.50	0.80	-0.15	1.00	1.00	1.00	373.32	371.76	371.74
0.50	0.50	1.20	-0.27	1.00	1.00	1.00	135.96	135.98	135.96
0.50	0.50	1.20	-0.35	1.00	1.00	1.00	34.34	34.31	34.31
0.50	0.50	1.60	-0.63	1.00	1.00	1.00	4.53	4.53	4.53
0.50	0.90	0.80	-0.15	1.00	1.00	1.00	684.28	618.18	618.04
0.50	0.90	1.20	-0.27	1.00	1.00	1.00	158.52	156.81	156.78
0.50	0.90	1.20	-0.35	1.01	1.00	1.00	43.53	42.18	42.17
0.50	0.90	1.60	-0.63	1.00	1.00	1.00	4.58	4.57	4.57
2.00	0.50	0.80	-0.15	1.00	1.00	1.00	624.90	575.01	575.11
2.00	0.50	1.20	-0.27	1.00	1.00	1.00	145.54	145.08	145.12
2.00	0.50	1.20	-0.35	1.01	1.00	1.00	38.82	38.35	38.35
2.00	0.50	1.60	-0.63	1.00	1.00	1.00	4.55	4.55	4.55
2.00	0.90	0.80	-0.15	1.04	1.00	1.00	2948.24	2010.89	1978.76
2.00	0.90	1.20	-0.27	1.01	1.00	1.00	244.81	224.22	224.28
2.00	0.90	1.20	-0.35	1.09	1.00	1.00	84.50	69.23	69.16
2.00	0.90	1.60	-0.63	1.04	1.01	1.01	4.79	4.75	4.75

APPENDIX C : Derivation of the asymptotic variances.

We will outline the derivation of the asymptotic variances of the estimators considered for model II. The results for models I and IV have been obtained along the same lines. Moreover we show that for model III all estimators considered are asymptotically efficient.

First we evaluate $E[y_{t+1} | I_{t-1}] = \alpha_{21}x_{t-1} + \alpha_{22}x_{t-2}$.

From (3.1) and (3.2) one has

$$\begin{aligned} E[y_{t+1} | I_{t-1}] &= \beta_1 E[y_{t+2} | I_{t-1}] + \beta_2 E[x_{t+1} | I_{t-1}] \\ &= \beta_1 \alpha_{21} E[x_t | I_{t-1}] + \beta_1 \alpha_{22} E[x_{t-1} | I_{t-1}] + \beta_2 E[x_{t+1} | I_{t-1}] \\ &= \{\beta_1 \alpha_{22} + \beta_1 \alpha_{21} \gamma_1 + \beta_2 (\gamma_1^2 + \gamma_2)\} x_{t-1} + \{\beta_1 \alpha_{21} \gamma_2 + \beta_2 \gamma_1 \gamma_2\} x_{t-2} \\ &= \alpha_{21} x_{t-1} + \alpha_{22} x_{t-2} \end{aligned} \quad (B1)$$

which yields

$$\begin{aligned} \alpha_{21} &= \beta_2 (\gamma_1^2 + \gamma_2 + \beta_1 \gamma_1 \gamma_2) (1 - \beta_1 \gamma_1 - \beta_1^2 \gamma_2)^{-1}, \\ \alpha_{22} &= \beta_2 \gamma_2 (\gamma_1 + \beta_1 \gamma_2) (1 - \beta_1 \gamma_1 - \beta_1^2 \gamma_2)^{-1}. \end{aligned} \quad (B2)$$

Using this result it is straightforward to evaluate $\text{plim } T^{-1} \hat{X}' \hat{X}, \text{plim } T^{-1} \hat{X}' Z$ and $\text{plim } T^{-1} \hat{X}' Z_{-1}$. The asymptotic variance of $\sqrt{T} \hat{\beta}_{\text{GLS}}$,

$$\text{var}(\sqrt{T} \hat{\beta}_{\text{GLS}}) = \text{plim } T^{-1} (\hat{X}' \hat{\Sigma}^{-1} \hat{X})^{-1} \quad (B3)$$

can now readily be evaluated using the expression in footnote 1). The asymptotic variance of $\hat{\beta}_{\text{OLS}}$ is

$$\begin{aligned} \text{var}(\sqrt{T} \hat{\beta}_{\text{OLS}}) &= \text{plim } T^{-1} (\hat{X}' \hat{X})^{-1} [\sigma^2 \hat{X}' \hat{X} + \beta_1^2 \hat{X}' Z (Z' Z)^{-1} G (Z' Z)^{-1} Z' \hat{X} \\ &\quad - \beta_1 \sigma^2 \hat{X}' Z (Z' Z)^{-1} Z_{-1}' \hat{X} - \beta_1 \sigma^2 \hat{X}' Z_{-1} (Z' Z)^{-1} Z' \hat{X}] (\hat{X}' \hat{X})^{-1} \end{aligned} \quad (B4)$$

with G being the asymptotic variance of $T^{-\frac{1}{2}} Z' u$ which can be evaluated using (3.3). For $\hat{\beta}_{\text{IV}}$ and $\hat{\beta}_{\text{CHO}}$ we find

$$\text{var}(\sqrt{T} \hat{\beta}_{\text{IV}}) = \text{plim } T^{-1} (Z^* \hat{X})^{-1} Z' \Omega Z (\hat{X}' Z^*)^{-1} \quad (B5)$$

and

$$\text{var}(\sqrt{T}\hat{\beta}_{\text{CHO}}) = \text{plim } T^{-1}[\tilde{X}'Z\tilde{\Omega}_Z^{-1}Z'\tilde{X}]^{-1} \quad (\text{B6})$$

respectively, where the notation is defined in section 2.

In order to derive the asymptotic variance of $\hat{\beta}_{\text{HS}}$ we first determine the MA(1) representation of \tilde{w} by equating the variance and first order serial correlation coefficient of $\tilde{w} = \varepsilon - \beta_1 u$ and $v_t + \theta v_{t-1}$ (with $v_t \cong \text{NID}(0, \sigma_v^2)$) and solving for θ and σ_v^2 . The asymptotic variance of $\hat{\beta}_{\text{HS}}$ is then given by

$$\text{var}(\sqrt{T}\hat{\beta}_{\text{HS}}) = \text{plim } T^{-1}\sigma_v^2[\tilde{X}'\tilde{\Omega}_v^{-1}Z(Z'Z)^{-1}Z'P^{-1}\tilde{X}]^{-1} \quad (\text{B7})$$

$$\text{with } \text{plim } T^{-1}\tilde{X}'P^{-1}Z = \text{plim } T^{-1}\sum_{i=0}^{\infty}(-\theta)^i\tilde{X}'_{+i}Z.$$

Finally $\text{var}(\sqrt{T}\hat{\beta}_{\text{ML}})$ has to be determined. Consider

$$y_t = \beta_2 x_t + \lambda_1 x_{t-1} + \lambda_2 x_{t-2} + \varepsilon_t \quad (\text{B8})$$

with $\lambda_1 = \beta_1 \alpha_{21}$ and $\lambda_2 = \beta_1 \alpha_{22}$. Denote the asymptotic variance of the unrestricted efficient estimator of $(\beta_1, \lambda_1, \lambda_2, \gamma_1, \gamma_2)$ by M^{-1} , and denote the matrix of first order derivatives of this vector with respect to the parameter vector of interest $(\beta_1, \beta_2, \gamma_1, \gamma_2)$ by S . $\text{var}(\sqrt{T}\hat{\beta}_{\text{ML}})$ is then the upper-left block of $\{SMS'\}^{-1}$.

Next we show that for model III all estimators are asymptotically efficient. First, consider

$$y_t = \lambda_1 x_t + \lambda_2 x_{t-1} + \varepsilon_t \quad (\text{B9})$$

with $\lambda_1 = \beta_1 \gamma_1 + \beta_2$ and $\lambda_2 = \beta_1 \gamma_2$. The maximum likelihood estimator $(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\gamma}_1, \hat{\gamma}_2)$ of $(\lambda_1, \lambda_2, \gamma_1, \gamma_2)$ is the equation by equation OLS estimator of (B9) and (3.2) respectively. As the transformation of this parameter vector to the parameter vector of interest $(\beta_1, \beta_2, \gamma_1, \gamma_2)$ is bijective, $\hat{\beta}_{\text{ML}}$ can be obtained from the inverse transformation as

$$\hat{\beta}_{1,\text{ML}} = \hat{\lambda}_2 \hat{\gamma}_2^{-1} \text{ and } \hat{\beta}_{2,\text{ML}} = \hat{\lambda}_1 - \hat{\lambda}_2 \hat{\gamma}_1 \hat{\gamma}_2^{-1}. \quad (\text{B10})$$

The estimator $\hat{\beta}_{\text{OLS}}$ is obtained from a regression of y_t on $\hat{\gamma}_1 x_t + \hat{\gamma}_2 x_{t-1}$ and x_t which is a linear transformation of the regressors in (B9).

Therefore $\hat{\beta}_{\text{OLS}}$ coincides with the corresponding inverse transformation of

$(\hat{\lambda}_1, \hat{\lambda}_2)$ which yields $\hat{\beta}_{OLS} = \hat{\beta}_{ML}$.

Along the lines of Kruskal (1968) it can simply be shown that $\hat{\beta}_{OLS} = \hat{\beta}_{GLS}$ as $\hat{\Sigma}\hat{X} = \hat{X}F$ for a non-singular matrix F . Moreover it can be directly verified that for this model $\hat{\beta}_{OLS} = \hat{\gamma}_{IV}$. Finally $\hat{\gamma}_{CHO}$ and $\hat{\gamma}_{HS}$ do not coincide with $\hat{\beta}_{OLS}$ but are asymptotically equally efficient as they are at least as efficient as $\hat{\gamma}_{IV}$.

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